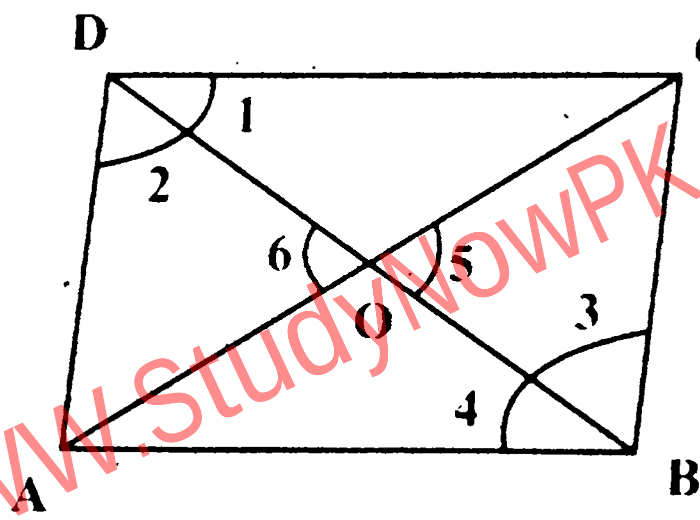
Unit 11
Parallelograms And Triangles

THEOREM 11.1.1

Prove that in a parallelogram:

(i) Opposite sides are congruent
(ii) Opposite angles are congruent.
(iii) The diagonals bisect each other.

Solution:



Given:

In a quadrilateral ABCD,

$\overline{BC} \parallel \overline{AD}$, $\overline{DC} \parallel \overline{AB}$ and the diagonals $\overline{AC}$, $\overline{BD}$ bisect each other at point O.

To Prove:

(i) $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$

(ii) $\angle BAD \cong \angle BCD$, $\angle ABC \cong \angle ADC$

(iii) $\overline{OB} \cong \overline{OD}$, $\overline{OA} \cong \overline{OC}$

Construction:

In the figure as shown, name the angles as:

$\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$

Proof:

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	A.S.A. $\cong$ A.S.A
So $\overline{AB} \cong \overline{DC} \cong \overline{AD} \cong \overline{BC}$	Corresponding sides of congruent triangles
And $\angle A \cong \angle C$	Corresponding angles of congruent triangles
(ii) In $\Delta ADB \leftrightarrow \Delta CDB$	
$\angle 1 \cong \angle 4$ .... (a)	Proved
$\angle 2 \cong \angle 3$ .... (b)	Proved
$m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$	From (a) and (b)
$\angle ADC \cong \angle ABC$	
Similarly $\angle BAD \cong \angle BCD$	
(iii) In $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$	Proved
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\therefore \Delta BOC \cong \Delta DOA$	A.A.S. $\cong$ A.A.S.
And $\overline{OC} \cong \overline{OA}$, $\overline{OB} \cong \overline{OD}$	Corresponding sides of congruent triangles

EXERCISE 11.1

Q1. One angle of a parallelogram is 130°. Find the measures of its remaining angles.

Solution:

In parallelogram ABCD

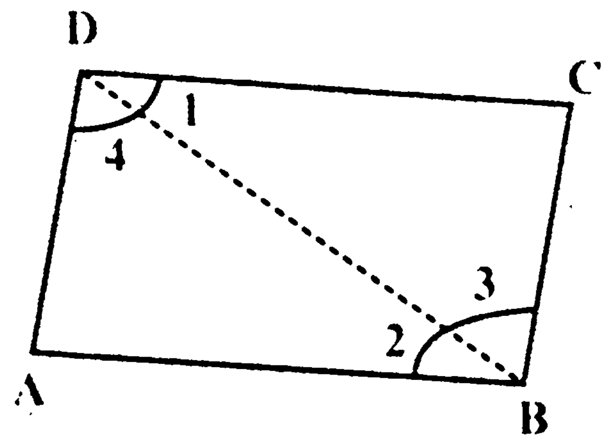
$m\angle B = 130°$

$\angle D \cong \angle B$

THEOREM 11.1.2

Prove that if two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.

Solution:



Given:

In a quadrilateral ABCD

$\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$

To Prove:

ABCD is a parallelogram

Construction:

Join the point B to D and in the figure name the angles as: $\angle 1, \angle 2, \angle 3,$ and $\angle 4$

Proof:

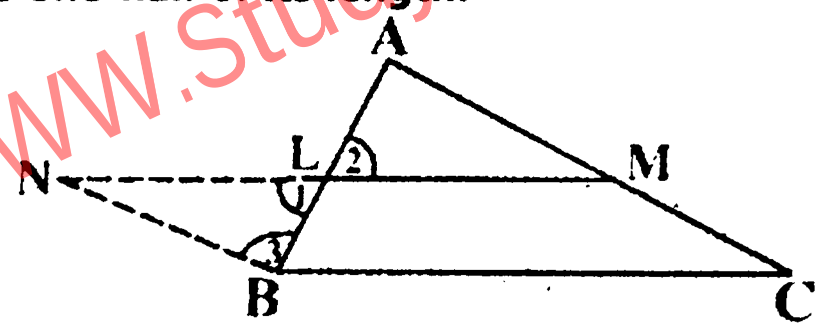
Statements	Reasons
$\angle 1 \cong \angle 2$	Alternate angles
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate Angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore$ $\Delta ABD \cong \Delta CDB$	S.A.S postulate
and $\angle 4 \cong \angle 3$ .... (i)	corresponding angles of congruent triangles
$\therefore$ $\overline{AD} \parallel \overline{BC}$ .... (ii)	From (i)
and $\overline{AD} \parallel \overline{DC}$ .... (iii)	Given
Thus ABCD is a parallelogram	From (ii) and (iii)

Proof:

Statements		Reasons
In $\Delta ABD \longleftrightarrow \Delta CBD$		
$\overline{AD} \cong \overline{CB}$		Given
$\overline{AB} \cong \overline{CD}$		Given
$\overline{BD} \cong \overline{DB}$		Common
$\therefore \Delta ABD \cong \Delta CDB$		S.S.S $\cong$ S.S.S.
$\angle 2 \cong \angle 1$	(i)	Corresponding angles of congruent triangles
and $\angle 4 \cong \angle 3$	(ii)	
Hence $\overline{AB} \parallel \overline{DC}$		(i) alternate angles
And $\overline{BC} \parallel \overline{AD}$		(ii) alternate angles
Hence ABCD is a parallelogram.		

THEOREM 11.1.3

The line segment that joins the mid-points of two sides of a triangle is parallel to the third side and is equal to one-half of its length.



Solution:

Given:

In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To Prove:

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction:

Join L to M and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$. Join N to B and in the figure, name the angles as: $\angle 1, \angle 2$ and $\angle 3$

Proof:

Statements	Reasons
In $\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \quad \Delta BLN \cong \Delta ALM$	S.A.S postulate
and $\angle A \cong \angle 3$ ...... (i)	Corresponding angles of congruent triangles
$\overline{NB} \cong \overline{AM}$ ...... (ii)	Corresponding sides of congruent triangles
$\overline{NB} \parallel \overline{AM}$	From (i)
$\Rightarrow \quad \overline{NB} \parallel \overline{MC}$ ...... (iii)	M is mid-point of $\overline{AC}$
$\overline{MC} \cong \overline{AM}$ ...... (iv)	Given
$\overline{NB} \cong \overline{MC}$ .... (v)	From (ii) and (iv)
$\therefore$ BCMN is a parallelogram	From (iii) and (v)
$\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	Opposite sides of a parallelogram BCMN
$\overline{BC} \cong \overline{MN}$ .... (vi)	Opposite sides of a parallelogram
$m\,\overline{LM} = \frac{1}{2}\,m\,\overline{NM}$ .... (vii)	Construction
Thus $m\,\overline{LM} = \frac{1}{2}\,m\,\overline{BC}$	From (vi) and (vii)

EXERCISE 11.3

Q1. Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:

Given:

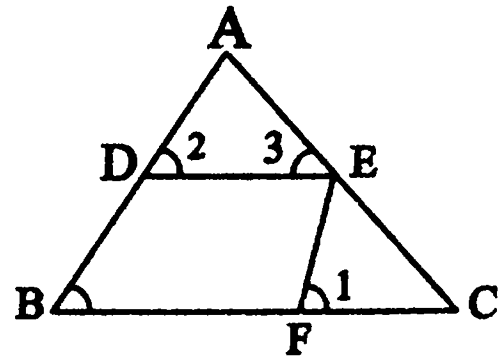
In quadrilateral ABCD, P, Q, R, S are the mid-points of the sides PR and QS are joined, they meet at O.

To prove:

$\overline{OP} \cong \overline{OR}$, $\overline{OQ} \cong \overline{OS}$

Q3. Prove that the line-segment passing through the mid-points of one side and parallel to another side of a triangle also bisect the third side.

Solution:



Given:

In ΔABC, D is mid-point of AB $\overline{DE} \parallel \overline{BC}$

To prove:

$\overline{EA} \cong \overline{EC}$

Construction:

Take $\overline{EF} \parallel \overline{AB}$

Proof:

Statements	Reasons
$\overline{DE} \parallel \overline{BF}$	Given
$\overline{EF} \parallel \overline{BD}$	Construction
$\therefore$ DBEF is a parallelogram.	
$\overline{EF} \cong \overline{DB}$ (i)	Opposite sides
$\overline{AD} \cong \overline{DB}$ (ii)	Given
$\overline{EF} \cong \overline{AD}$ (iii)	From (i) and (ii)
$\angle 1 \cong \angle B$	corresponding angles
and $\angle 2 \cong \angle B$	
$\therefore \angle 1 \cong \angle 2$ (iv)	
In $\Delta ADE \longleftrightarrow \Delta EFC$	
$\angle 2 \cong \angle 1$	From (iv)
$\therefore \angle 3 \cong \angle C$	Corresponding angle
$\overline{AD} \cong \overline{EF}$	From (iii)
Hence $\Delta ADE \cong \Delta EFC$	A.A.S $\cong$ A.A.S.
$\therefore \overline{EA} \cong \overline{EC}$	Corresponding sides

THEOREM 11.1.4

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.

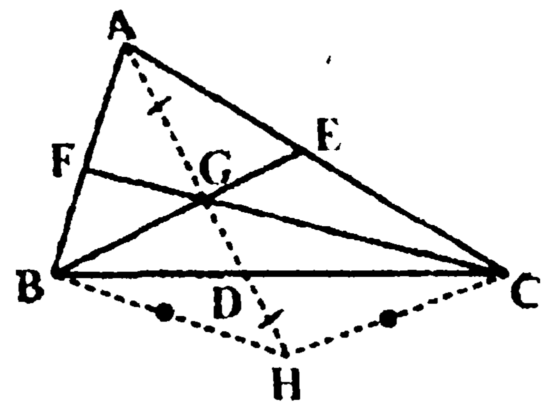
Solution:

Given:

ABC is a triangle

To Prove:

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median.



Construction:

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to point H such that $\overline{AG} \cong \overline{GH}$. Join H to the points B and C. D is the intersecting point of $\overline{AH}$ and $\overline{BC}$.

Proof:

Statements	Reasons
In ΔACH, $\overline{GE} \parallel \overline{CH}$	$\because$ E and G are the midpoints of $\overline{AC}$ and $\overline{AH}$
Or $\overline{BE} \parallel \overline{CH}$ .... (i)	G is a point of $\overline{BE}$
Similarly $\overline{CF} \parallel \overline{BH}$ ..... (ii)	From (i)
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
and $m\overline{GD} = \frac{1}{2} m\overline{GH}$..... (iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram BHCG intersect each other at point D.
$\overline{AD}$ is a median of ΔABC Medians $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ pass through point G	G is the intersecting point of $\overline{BE}$ and $\overline{CF}$ and $\overline{AD}$ pass through it
Now $\overline{GH} \cong \overline{AG}$ ...... (iv)	Construction
$\therefore$ $m\overline{GD} \cong \frac{1}{2} m\overline{AG}$ and G is the point of trisection of AD .... (v) Similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	From (iii) and (iv)

To prove:

G is the point of concurrency of the medians of ΔABC and ΔPQR.

Proof:

Statements	**Reasons**
$\overline{PR} \parallel \overline{BC}$ $\overline{PR} \parallel \overline{BQ}$ Similarly $\overline{QR} \parallel \overline{BP}$ $\therefore$ PBQR is a parallelogram. Its diagonal $\overline{BR}$ and $\overline{PQ}$ bisect each other at T. i.e. T is mid-point of $\overline{PQ}$. Similarly U is mid-point of $\overline{QR}$ and S is mid-point of $\overline{PR}$. $\therefore$ $\overline{PU}$, $\overline{QS}$, $\overline{RT}$ are medians of ΔPQR (i) $\overline{AQ}$ and $\overline{SQ}$ pass through G. (ii) $\overline{BR}$ and $\overline{TR}$ pass through G. (iii) $\overline{CP}$ and $\overline{UP}$ pass through G. Hence G is point of concurrency of medians of ΔAGC and ΔPQR.	P, R are mid-points of $\overline{AB}$, $\overline{AC}$.

THEOREM 11.1.5

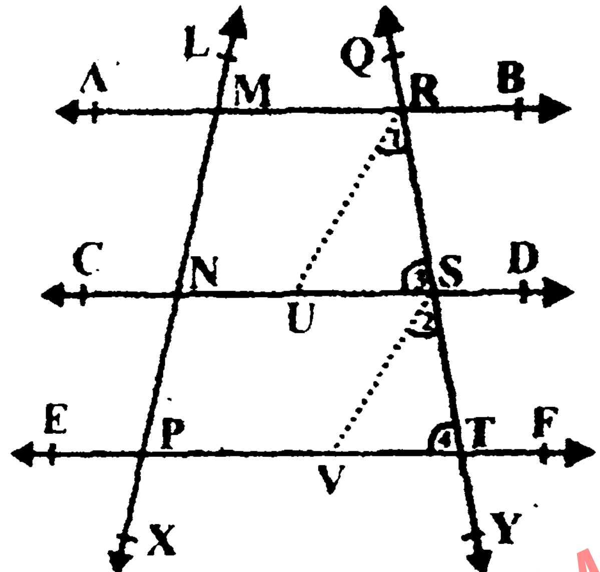
If three or more parallel lines make segments congruent on one transversal, they also make congruent segments on any other transversal.

Solution:

Given:

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

$\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points **M, N** and P respectively, such that $\overleftrightarrow{MN} \cong \overleftrightarrow{NP}$. $\overleftrightarrow{QY}$ intersects them at points R,S and T respectively.



To Prove:

$\overline{RS} \cong \overline{ST}$

Construction:

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V and according to the figure the names of the angles are $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	Reasons
MNUR is a parallelogram	$\overline{RU} \parallel \overline{LX}$ (construction) $\overline{AB} \parallel \overline{CD}$ (given)
$\therefore$ $\overline{MN} \cong \overline{RU}$ .... (i)	Opposite sides of parallelogram
Similarly $\overline{NP} \cong \overline{SV}$ .... (ii)	
But $\overline{MN} \cong \overline{NP}$ .... (iii)	Given
$\therefore$ $\overline{RU} \cong \overline{SV}$	From (i), (iii) and (iii)
Also $\overline{RU} \parallel \overline{SV}$	Each one $\parallel \overleftrightarrow{LX}$ (construction)
$\therefore$ $\angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$,	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore$ $\Delta RUS \cong \Delta SVT$	S.A.A $\cong$ S.A.A
And $\overline{RS} \cong \overline{ST}$	Corresponding sides of congruent triangles

Proof:

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	A.S.A. ≅ A.S.A
So $\overline{AB} \cong \overline{DC} \cong \overline{AD} \cong \overline{BC}$	Corresponding sides of congruent triangles
And $\angle A \cong \angle C$	Corresponding angles of congruent triangles
(ii) In $\Delta ADB \leftrightarrow \Delta CDB$	
$\angle 1 \cong \angle 4$ .... (a)	Proved
$\angle 2 \cong \angle 3$ .... (b)	Proved
$m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$	From (a) and (b)
$\angle ADC \cong \angle ABC$	
Similarly $\angle BAD \cong \angle BCD$	
(iii) In $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$.	Proved
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\therefore \Delta BOC \cong \Delta DOA$	A.A.S. ≅ A.A.S.
And $\overline{OC} \cong \overline{OA}$, $\overline{OB} \cong \overline{OD}$	Corresponding sides of congruent triangles

EXERCISE 11.1

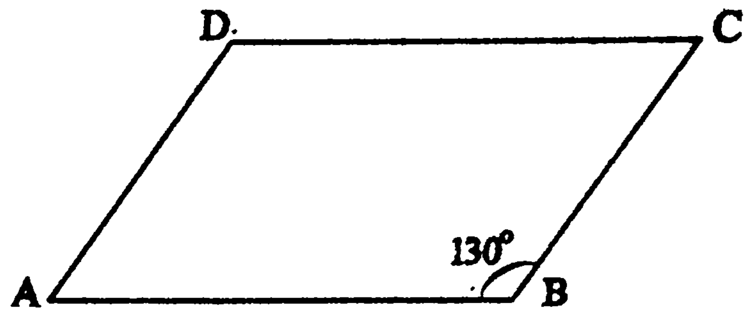
Q1. One angle of a parallelogram is 130°. Find the measures of its remaining angles.

Solution:

In parallelogram ABCD

$m\angle B = 130°$

$\angle D \cong \angle B$



Opposite angles of a parallelogram

$\therefore$ $m\angle D = m\angle B = 130°$

$m\angle B + m\angle A = 180°$

$130° + m\angle A = 180°$

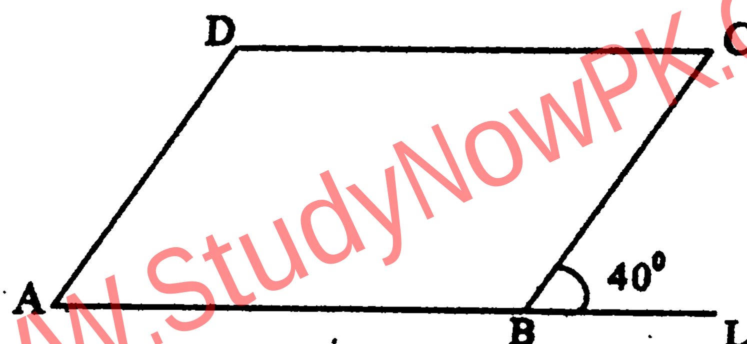
$m\angle A = 180° - 130° = 50°$

$m\angle C = m\angle A = 50°$

So unknown angles of parallelogram are 130°, 50° and

Q2. One exterior angle formed on producing one side of a parallelogram is 40°.

Solution:



ABCD is a parallelogram $m\angle CBL = 40°$

$m\angle ABC + 40° = 180°$

$\therefore$ ABL is a straight line

$\therefore$ $m\angle ABC = 180° - 40° = 140°$

$m\angle D = m\angle ABC = 140°$

Opposite angles of a parallelogram

$m\angle D + m\angle C = 180°$

$140° + m\angle C = 180°$

$\therefore$ $m\angle C = 180° - 140° = 40°$

$m\angle A = m\angle C = 40°$

Opposite angles of parallelogram

So the measures of interior angles of the parallelogram are 140°, 40°, 140° and 40°.

EXERCISE 11.2

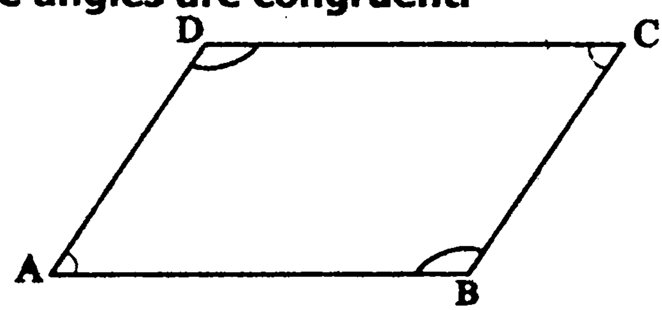
Q1.a) Prove that a quadrilateral is a parallelogram if its

(a) Opposite angles are congruent.

(b) Diagonals bisect each other.

Solution:

(a) Opposite angles are congruent.



Given:

In quadrilateral ABCD

$m\angle A = m\angle C,$

$m\angle B = m\angle D$

To prove:

ABCD is a parallelogram

Proof:

Statements	Reasons
$m\angle A = m\angle C$ (i)	Given
$m\angle B = m\angle D$ (ii)	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360^0$	Angles of quadrilateral
$m\angle A + m\angle B + m\angle A + m\angle B = 360^0$	From (i) and (ii)
$2m\angle A + 2m\angle B = 360$	
$\therefore m\angle A + m\angle B = 180$	
$\therefore \overline{AD} \parallel \overline{BC}$	Sum of internal angles
Similarly it can be proved that $\overline{AB} \parallel \overline{CD}$	
Hence ABCD is a parallelogram.	

(b) Diagonals bisect each other.

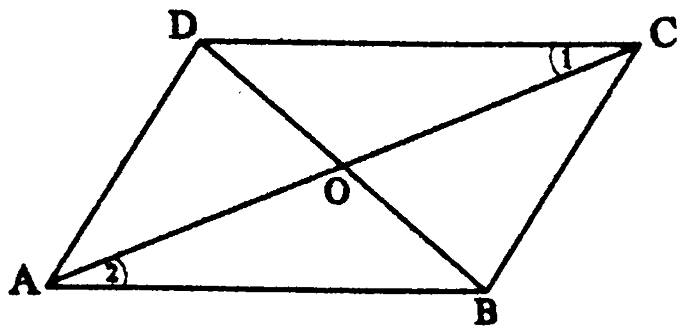
Solution:

Given:

In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other. i.e. $\overline{OA} = \overline{OC}$, $\overline{OB} = \overline{OD}$

To prove:

ABCD is a parallelogram

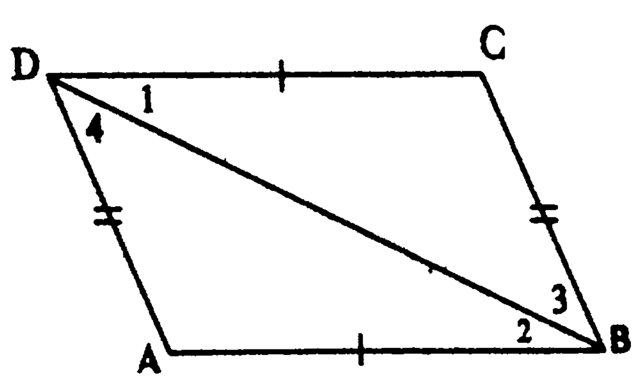


Proof:

Statements	Reasons
In $\Delta ABO \longleftrightarrow \Delta CDO$	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \Delta AOB \cong \angle COD$	S.A.S ≅ S.A.S.
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
Hence $\overline{AB} \parallel \overline{OC}$ (i)	$\angle 1 \cong \angle 2$
By taking Δ 's AOD and BOC we can prove that $\overline{AD} \parallel \overline{BC}$ (ii)	
Hence ABCD is a parallelogram.	From (i) and (ii)

Q2. Prove that a quadrilateral is a parallelogram if its opposite sides are congruent.

Solution:



Given:

In quadrilateral ABCD

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

To prove:

ABCD is a parallelogram

Construction:

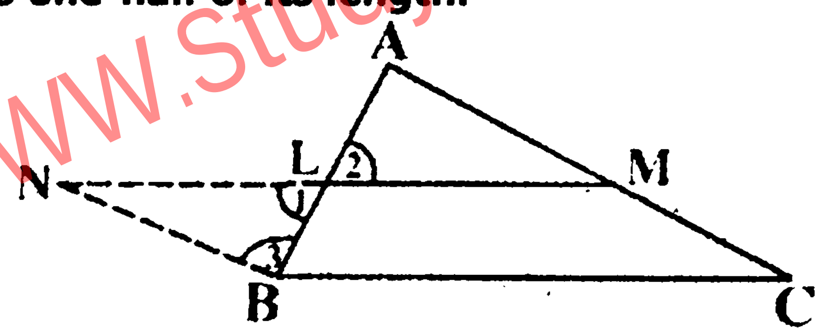
Join B to D

Proof:

Statements	Reasons
In $\Delta ABD \longleftrightarrow \Delta CBD$	
$\overline{AD} \cong \overline{CB}$	Given
$\overline{AB} \cong \overline{CD}$	Given
$\overline{BD} \cong \overline{DB}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	S.S.S $\cong$ S.S.S.
$\angle 2 \cong \angle 1$ (i)	Corresponding angles of congruent triangles
and $\angle 4 \cong \angle 3$ (ii)	(i) alternate angles
Hence $\overline{AB} \parallel \overline{DC}$	(ii) alternate angles
And $\overline{BC} \parallel \overline{AD}$	
Hence ABCD is a parallelogram.	

THEOREM 11.1.3

The line segment that joins the mid-points of two sides of a triangle is parallel to the third side and is equal to one-half of its length.



Solution:

Given:

In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To Prove:

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction:

Join L to M and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$. Join N to B and in the figure, name the angles as: $\angle 1, \angle 2$ and $\angle 3$

Proof:

Statements	Reasons
In $\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \quad \Delta BLN \cong \Delta ALM$	S.A.S postulate
and $\angle A \cong \angle 3$ ...... (i)	Corresponding angles of congruent triangles
$\overline{NB} \cong \overline{AM}$ ...... (ii)	Corresponding sides of congruent triangles
$\overline{NB} \parallel \overline{AM}$	From (i)
$\Rightarrow \quad \overline{NB} \parallel \overline{MC}$ ...... (iii)	M is mid-point of $\overline{AC}$
$\overline{MC} \cong \overline{AM}$ ...... (iv)	Given
$\overline{NB} \cong \overline{MC}$ .... (v)	From (ii) and (iv)
$\therefore$ BCMN is a parallelogram	From (iii) and (v)
$\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	Opposite sides of a parallelogram BCMN
$\overline{BC} \cong \overline{MN}$ .... (vi)	Opposite sides of a parallelogram
$m\,\overline{LM} = \frac{1}{2}\,m\,\overline{NM}$ .... (vii)	Construction
Thus $m\,\overline{LM} = \frac{1}{2}\,m\,\overline{BC}$	From (vi) and (vii)

EXERCISE 11.3

Q1. Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:

Given:

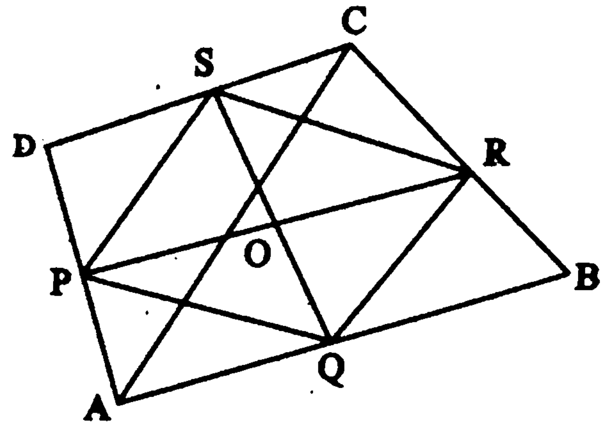
In quadrilateral ABCD, P, Q, R, S are the mid-points of the sides PR and QS are joined, they meet at O.

To prove:

$\overline{OP} \cong \overline{OR}$, $\overline{OQ} \cong \overline{OS}$

Construction:

Join P, Q, R, S in order. Join A to C.



Proof:

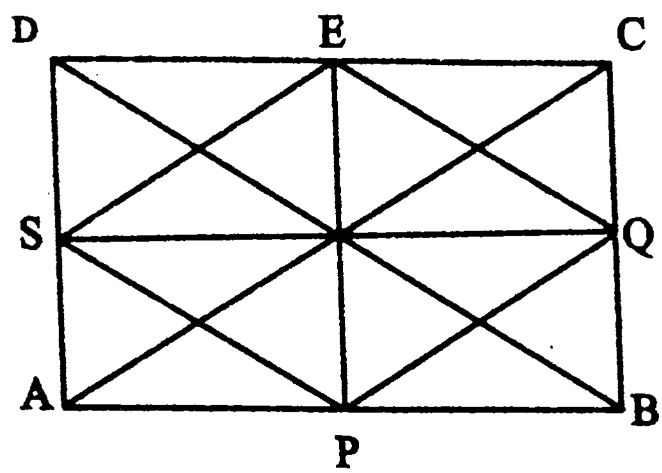
Statements	Reasons
$\overline{SD} \cong \overline{AC}$ (i)	In Δ ADC, S, P are mid-points of AD, DC.
$m\overline{SP} = \frac{1}{2}m\overline{AC}$ (ii)	In Δ ABC, P, Q are mid-points of AB, BC.
$\overline{RQ} \parallel \overline{AC}$ (iii)	
$m\overline{RQ} = \frac{1}{2}m\overline{AC}$ (iv)	
$\therefore$ $\overline{SP} \parallel \overline{RQ}$ (v)	From (ii) and (iv)
and $m\overline{RQ} = \frac{1}{2}m\overline{AC}$ (vi)	
$\therefore$ PQRS is a parallelogram	From (v) and (vi)
Now $\overline{PR}$ and $\overline{SQ}$ diagonals of parallelogram PQRS intersect at O	
$\therefore$ $\overline{OP} \cong \overline{OR}$	Diagonals of a parallelogram bisect each other
And $\overline{OS} \cong \overline{OQ}$	

Q2. Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.

Solution:

Given:

In rectangle ABCD, P, Q, R, S are mid-point of the sides P is joined to R, Q is joined to S. $\overline{PR}$ and $\overline{QS}$ intersect at O.



To prove:

$\overline{PR}$ and $\overline{QS}$ are right bisectors of each other.

Construction:

Join P, Q, R, S in order. Join A to C and B to D.

Proof:

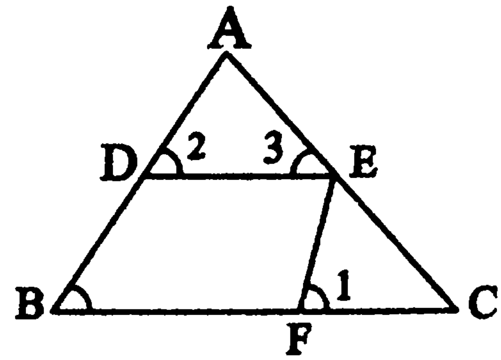
Statements		Reasons
$\overline{SR} \parallel \overline{AC}$	(i)	In Δ ADC, S, R are mid-points of AD, DC.
$m\overline{SR} = \frac{1}{2} m\overline{AC}$	(ii)	
and $\overline{PQ} \parallel \overline{AC}$	(iii)	In Δ ABC, P, Q are mid-points of $\overline{AB}$, $\overline{BC}$.
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$	(iv)	
$\therefore \overline{SR} \parallel \overline{PQ}$	(v)	From (i) and (iii)
$m\overline{SR} = m\overline{PQ}$	(vi)	From (ii) and (iv)
$\therefore$ PQRS is a parallelogram		From (v) and (vi)
$m\overline{AC} = m\overline{BD}$		Diagonals of a rectangle
$\frac{1}{2} m\overline{AC} = \frac{1}{2} m\overline{BD}$		
$m\overline{PQ} = m\overline{QR}$		
$\therefore m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{SP}$		
$\therefore$ PQRS is a rhombus.		
$\overline{PR}$ and $\overline{QS}$ are diagonals of rhombus PQRS.		
$\therefore$ $\overline{PR}$ and $\overline{QS}$ are right bisectors of each other.		Diagonals of a rhombus are right bisector of each other.

Q3. Prove that the line-segment passing through the mid-points of one side and parallel to another side of a triangle also bisect the third side.

Solution:

Given:

In ΔABC, D is mid-point of AB $\overline{DE} \parallel \overline{BC}$



To prove:

$\overline{EA} \cong \overline{EC}$

Construction:

Take $\overline{EF} \parallel \overline{AB}$

Proof:

Statements	Reasons
$\overline{DE} \parallel \overline{BF}$	Given
$\overline{EF} \parallel \overline{BD}$	Construction
$\therefore$ DBEF is a parallelogram.	
$\overline{EF} \cong \overline{DB}$ (i)	Opposite sides
$\overline{AD} \cong \overline{DB}$ (ii)	Given
$\overline{EF} \cong \overline{AD}$ (iii)	
$\angle 1 \cong \angle B$	From (i) and (ii)
and $\angle 2 \cong \angle B$	corresponding angles
$\therefore \angle 1 \cong \angle 2$ (iv)	
In $\Delta ADE \longleftrightarrow \Delta EFC$	
$\angle 2 \cong \angle 1$	From (iv)
$\therefore \angle 3 \cong \angle C$	Corresponding angle
$\overline{AD} \cong \overline{EF}$	From (iii)
Hence $\Delta ADE \cong \Delta EFC$	A.A.S $\cong$ A.A.S.
$\therefore \overline{EA} \cong \overline{EC}$	Corresponding sides

THEOREM 11.1.4

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.

Solution:

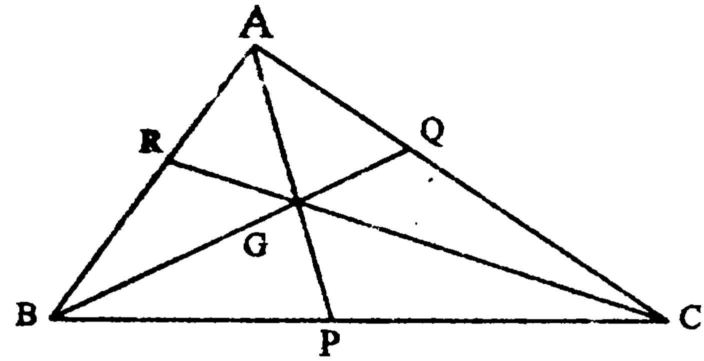
Given:

ABC is a triangle

EXERCISE 11.4

The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm, 1.4 cm and 1.6 cm. Find the lengths of its medians.

Solution:



Let ABC be triangle with the point of concurrency of medians at G.

$m\overline{AG} = 1.2$ cm, $m\overline{BG} = 1.4$ cm and $m\overline{CG} = 1.6$ cm

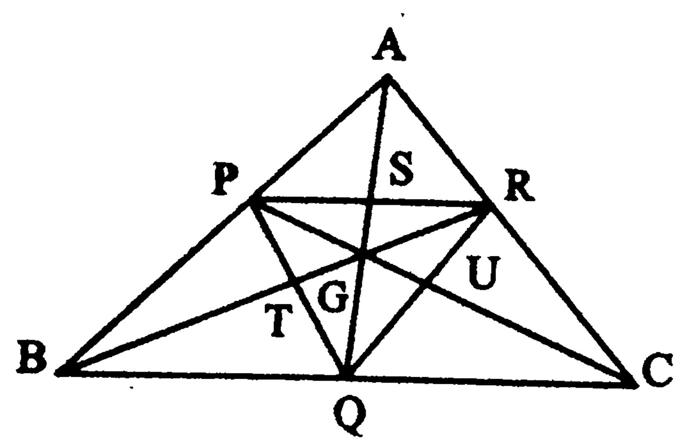
$m(\overline{AP}) = \frac{3}{2}(m\overline{AG}) = \frac{3}{2} \times 1.2 = 1.8$ cm

$m\overline{BQ} = \frac{3}{2}(m\overline{BG}) = \frac{3}{2} \times 1.4 = 2.1$ cm

$m\overline{CR} = \frac{3}{2}(m\overline{CG}) = \frac{3}{2} \times 1.6 = 2.4$ cm

Q2. Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the mid-points of its sides is the same.

Solution:



Given:

In triangle ABC, $\overline{CP}$, $\overline{AQ}$, $\overline{BR}$ are medians, with meet at G. ΔPQR is formed by joining the mid points P, Q, R.

To prove:

G is the point of concurrency of the medians of ΔABC and ΔPQR.

Proof:

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$ $\overline{PR} \parallel \overline{BQ}$ Similarly $\overline{QR} \parallel \overline{BP}$ $\therefore$ PBQR is a parallelogram. Its diagonal $\overline{BR}$ and $\overline{PQ}$ bisect each other at T. i.e. T is mid-point of $\overline{PQ}$. Similarly U is mid-point of $\overline{QR}$ and S is mid-point of $\overline{PR}$. $\therefore$ $\overline{PU}$, $\overline{QS}$, $\overline{RT}$ are medians of ΔPQR (i) $\overline{AQ}$ and $\overline{SQ}$ pass through G. (ii) $\overline{BR}$ and $\overline{TR}$ pass through G. (iii) $\overline{CP}$ and $\overline{UP}$ pass through G. Hence G is point of concurrency of medians of ΔAGC and ΔPQR.	P, R are mid-points of $\overline{AB}$, $\overline{AC}$.

THEOREM 11.1.5

If three or more parallel lines make segments congruent on one transversal, they also make congruent segments on any other transversal.

Solution:

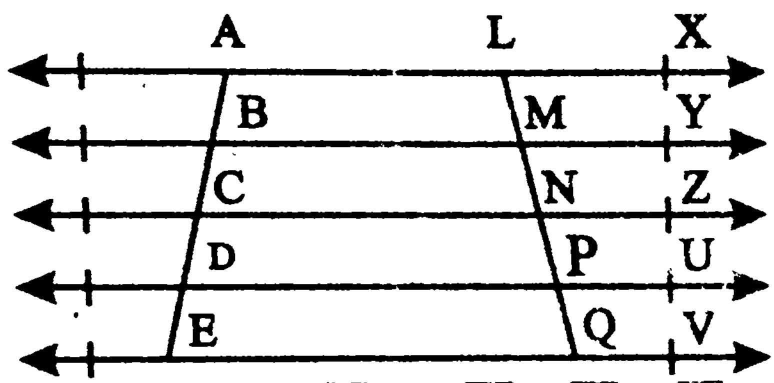
Given:

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

EXERCISE 11.5

Q1. In the given figure, $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$. If $m\overline{MN} = 1$ cm, then find the length of $\overline{LN}$ and $\overline{LQ}$.

Solution:



$\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DU} \parallel \overleftrightarrow{EV}$ and $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$

$\overline{LM} \cong \overline{MN} \cong \overline{NP} \cong \overline{PQ}$

$m\,\overline{LN} = m\,\overline{LM} + m\,\overline{MN}$

$= m\,\overline{MN} + m\,\overline{MN}$

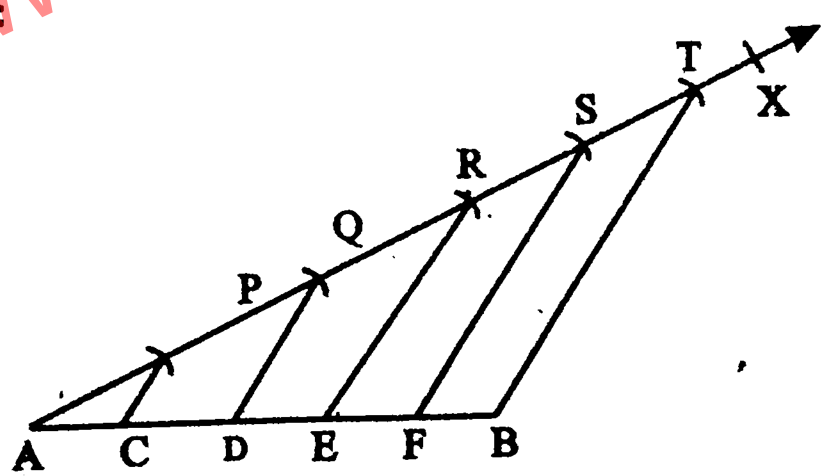
$= 1\text{ cm} + 1\text{ m} = 2\text{ cm}$

$m\overline{LQ} = m\overline{LM} + m\overline{MN} + n\overline{NP} + m\overline{PQ}$

$= 1\text{cm} + 1\text{ cm} + 1\text{ cm} + 1\text{ cm} = 4\text{ cm}$

Q2. Take a line segment of length 5.5 cm and divide it into five congruent parts.

Solution:



Construction:

(i) Draw a line segment $\overline{AB}$ of length 5 cm.

(ii) Draw an acute angle $\angle BAX$.

(iii) On $\overrightarrow{AX}$ with the help of compass take five points P, Q, R, S, T such that $\overline{AP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST}$.

(iv) Join T to B.
(v) Draw lines $\overline{SF}$, $\overline{RE}$, $\overline{QD}$, $\overline{PC}$ parallel to $\overline{TB}$.
The points C, D, E, F divide the line segment $\overline{AB}$ into five congruent parts.

REVIEW EXERCISE 11

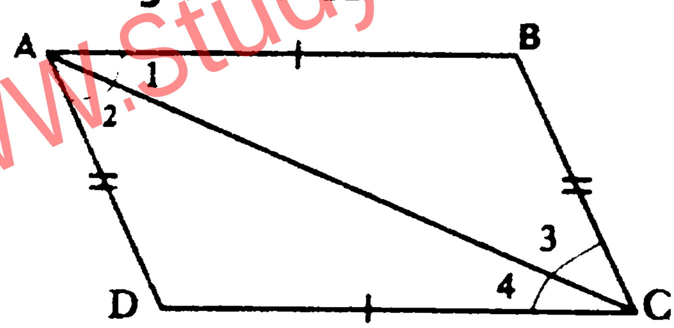
Q1. Fill in the blanks.

(i) In a parallelogram opposite sides are ....
(ii) In a parallelogram opposite angles are ....
(iii) Diagonals of a parallelogram .... each other at a point.
(iv) Medians of a triangle are ....
(v) Diagonals of a parallelogram divides the parallelogram into two .... triangles.

Answers:

(i) **parallel/congruent** (ii) **equal/congruent**
(iii) **intersect** (iv) **concurrent**
(v) **congruent**

Q2. In parallelogram ABCD



(i) $m\overline{AB}$ ..... $m\overline{DC}$ (ii) $m\overline{BC}$ ..... $m\overline{AD}$
(iii) $m\angle 1 \cong$ ..... (iv) $m\angle 2 \cong$ .....

Answers:

(i) $\cong$ (ii) $\cong$
(iii) $m\angle 3$ (iv) $m\angle 1$

Q3. Find the unknowns in the given figure.

Solution:

$n^\circ \cong 75^\circ$ opposite angles are congruent
$n = 75$
$y^\circ \cong n^\circ$ Alternate angles
$y^\circ \cong n^\circ \cong 75^\circ$

(iv) Join T to B.

(v) Draw lines $\overline{SF}$, $\overline{RE}$, $\overline{QD}$, $\overline{PC}$ parallel to $\overline{TB}$.
The points C, D, E, F divide the line segment $\overline{AB}$ into five congruent parts.

REVIEW EXERCISE 11

Q1. Fill in the blanks.

(i) In a parallelogram opposite sides are ....

(ii) In a parallelogram opposite angles are ....

(iii) Diagonals of a parallelogram .... each other at a point.

(iv) Medians of a triangle are ....

(v) Diagonals of a parallelogram divides the parallelogram into two .... triangles.

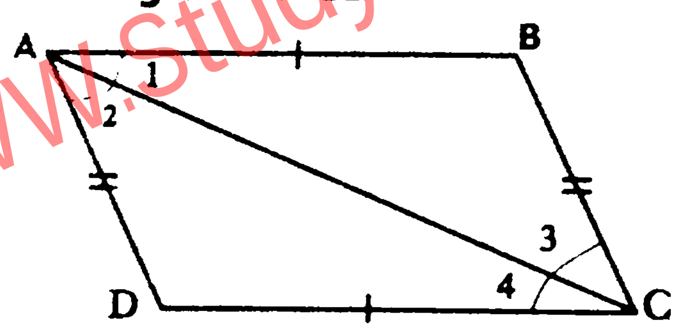
Answers:

(i) **parallel/congruent** **(ii)** **equal/congruent**

(iii) **intersect** **(iv)** **concurrent**

(v) **congruent**

Q2. In parallelogram ABCD



(i) $m\overline{AB}$ ..... $m\overline{DC}$ (ii) $m\overline{BC}$ ..... $m\overline{AD}$

(iii) $m\angle 1 \cong$ ..... (iv) $m\angle 2 \cong$ .....

Answers:

(i) $\cong$ (ii) $\cong$

(iii) $m\angle 3$ (iv) $m\angle 1$

Q3. Find the unknowns in the given figure.

Solution:

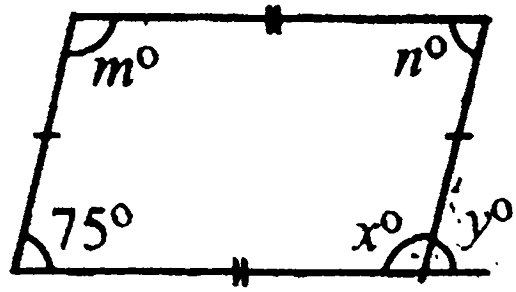
$n^\circ \cong 75^\circ$ opposite angles are congruent

$n = 75$

$y^\circ \cong n^\circ$ Alternate angles

$y^\circ \cong n^\circ \cong 75^\circ$

$y = 45$



$x^\circ + y^\circ = 180^\circ$ Supplementary angles

$x + y = 180$

$x + 75 = 180$

$x = 180 - 75 = 105$

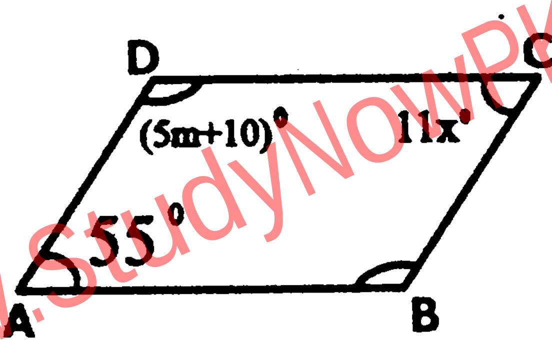
$m^\circ \cong x^\circ$ opposite angles

$m = x = 105$

$m^\circ = 105^0$

Q4. If the given figure ABCD is a parallelogram, then find x, m.

Solution:



$11x^b \cong 55^\circ$ opposite angles

$11x = 55$

$x = 5^0$

$(5m + 10)^\circ + 55^\circ = 180^\circ$

Sum of interior angles of ‖ lines

$5m + 10 + 55 = 180$

$5m + 65 = 180$

or $m = 23^0$

Q5. The given figure LMNP is a parallelogram. Find the value of m, n.

Solution:

As opposite sides of a parallelogram are congruent

$8m - 4n = 8$

or $2m - n = 2$ (i)

and $4m + n = 10$ (ii)

Adding (i) and (ii)

$6m = 12$

or $m = 2$

Putting $m = 2$ in (i)

we have

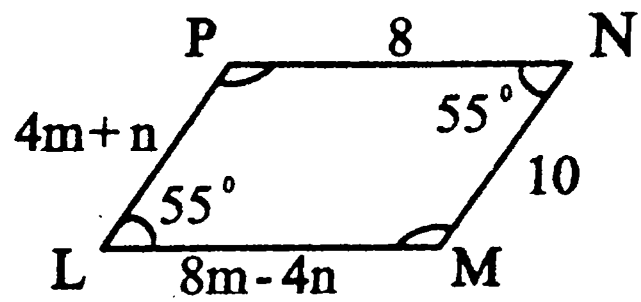
$2(2) - n = 2$

$4 - n = 2$

or $-n = -2$

or $n = 2$

$\therefore$ **$m = 2$, $n = 2$**



Q6. In the question 5, sum of the opposite angles of the parallelogram is 110°, find the remaining angles.

Solution:

Opposite angles of a parallelogram are congruent

$\angle L \cong \angle N$

But it is given that

$m\angle L + m\angle N = 110$

$2(m\angle L) = 110$

$m\angle L = 55$

$m\angle L = m\angle N = 55°$

$m\angle L + m\angle P = 180°$

Sum of interior angles between parallel lines

$55 + m\angle P = 180°$

$m\angle P = 180^0 - 55^0 = 125^0$

Angles of the parallelogram are

55°, 125°, 55° and 125°

$m\angle M = m\angle P = 125°$